Whitepaper

Belt Conveyor Technology

The Importance of Nonlinear Strain Considerations in Belt Cover Indentation Loss

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Authors:

T.J. Rudolphi, Department of Aerospace Engineering, Iowa State University, Ames, IA, USA
A. Reicks, Overland Conveyor Co., Lakewood, CO, USA

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T. J. Rudolphi and A.V. Reicks*

Abstract:

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Keywords: Belt conveying, indentation loss, rolling resistance, rubber property characterization, nonlinear stress-strain, Payne effect, Kraus' model.

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* Thomas J. Rudolphi, Department of Aerospace Engineering, Iowa State University, Ames, IA, USA
Al Reicks, Overland Conveyor Co., Lakewood, CO, USA
Email: rudolphi@iastate.edu

1. Introduction

Modern conveyor designs rely on predictive and analytical models of the rolling resistance due to indentation of idlers into the belt backing or cover material. A typical belt layup includes two rubber covers with a cord ply or steel cable carcass sandwiched between, with the top cover designed to resist wear and abrasion from the carried materials and the bottom to minimize rolling resistance over the belt system idlers. The indentation of the bottom cover by idlers is a primary source of power loss, and hence an important factor in the design of the system.

The particular characteristic of rubber compounds, especially those with carbon black fillers, that produces a nonlinear stress-strain behavior, even for relatively small strains, is called the Payne effect. This phenomenon refers to a dependence of the viscoelastic moduli on applied cyclic strain amplitude, where it is observed that for strains above about 0.1%, the storage modulus decreases as the loss modulus increases (cf. refs. [1-8]). As strains experienced by belt covers, even under modest loads, can routinely exceed 0.1%, and both the storage and loss moduli are factors in the indentation loss prediction, it is important to account for the effect of strain amplitude in any predictive model.

In this study we use the measured viscoelastic properties of a typical backing rubber material and fit that data to an established strain amplitude dependent constitutive model. The constitutive model is then incorporated into a one dimensional model of the indentation resistance to show that the indentation loss factor depends significantly on the strain levels and hence the loads carried by the belt.

2. Rubber properties and material characterization

Since characterization of nonlinear viscoelastic materials is an extension of the linear theory, we first review the linear methodology. As dissipative materials are inherently time dependent, testing in a cyclic mode at various frequencies is a natural way to capture the time dependence through Fourier analysis. For a sinusoidal strain history of the form $\varepsilon = \varepsilon_0 \sin(\omega t)$, where ω is the angular frequency and ε_0 the amplitude, after an initial transitory state the stress follows the strain in frequency, but at a delayed phase or angle δ. For a linear viscoelastic material, the one-dimensional stress/strain relationship is of the form,

$$\sigma(\omega) = E^*(\omega)\varepsilon_0 \sin(\omega t + \delta) \tag{1}$$

where σ denotes the stress, $E^*(\omega) = \sqrt{E'(\omega)^2 + E''(\omega)^2}$ is the magnitude of the complex modulus with real and imaginary components $E'(\omega)$ and $E''(\omega)$, called the storage and loss moduli, respectively. For viscoelastic materials undergoing harmonic deformation, the loss tangent is determined by $\tan(\delta) = E''/E'$ and is the phase angle of stress following strain.

By testing over a wide temperature range but limited frequencies, the superposition principle (cf. Weinman, et. al. [9] or Ferry [10]) of linear viscoelasticity, allows extrapolation to frequencies or load rates considerably beyond those possible in testing. According to this principle, if T denotes temperature and T_0 a reference temperature, then a material property at time t and temperature T is equivalent to that at time $a_T t$ and temperature T_0, where a_T is a shift parameter in time so that $a_T = a_T(T, T_0)$. A commonly used expression for a_T, applicable to amorphous materials such as rubbers, at temperatures above the glass transition temperature, due to Williams, Landel and Ferry [11] and referred to as the WLF equation, is the logarithmic form,

$$\log(a_T) = \frac{C_1(T - T_0)}{C_2 + (T - T_0)} \tag{2}$$

where C_1 and C_2 are constants determined empirically from data. For data taken from cyclical or frequency controlled strain experiments, an empirical representation of the shift parameter a_T may be determined by overlaying, through shifts in the frequency, the temperature dependent modulus data onto a "master" curve and software to do this shifting is usually a part of the DMA instrumentation or can be written according to various curve fitting or error minimization algorithms.

Various arrangements of linear mechanical elements of springs and dashpots are classically used to fit the viscoelastic behavior of eqn. (1). The simple Maxwell model consists of a single spring (solid) in parallel with another spring and a dashpot (fluid). A generalization of this, called a generalized Maxwell, or Weichert model [9], consists of many such springs and dashpots in parallel with one spring. For such an arrangement of N elements, the storage and loss moduli of eqn. (1) are related to the mechanical element values by the Prony series,

$$E'(\omega) = E_0 + \sum_{i=1}^{N} E_i \frac{\omega^2 \tau_i^2}{1 + \omega^2 \tau_i^2} \qquad\qquad E''(\omega) = \sum_{i=1}^{N} E_i \frac{\omega \tau_i}{1 + \omega^2 \tau_i^2} \tag{3}$$

where E_i and η_i are the discrete spring and dashpot constants and $\tau_i = \eta_i / E_i$ are the periods of the various spring/dashpot elements. The E_i values, as a set, are also called the spectrum, or spectral moduli, of the storage and loss moduli of eqn. (3), respectively.

As a shear mode of deformation is often used in testing, the equivalent shear storage modulus $G'(\omega)$ and loss modulus $G''(\omega)$ are convenient to use, instead of the tensile moduli of eqn. (3), with $G'(\omega) = E'(\omega)/3$ and $G''(\omega) = E''(\omega)/3$, assuming incompressibility of the rubber.

A computer program, based on least square curve fitting for shifting and overlaying test data at various temperatures, was written to determine the a_T vs. temperature relationship. Also, a least squares fitting process determines constants C_1 and C_2 of eqn. (2) and a least square, non-negative fitting process determines the spectral values of the moduli of eqn. (3). Here we determined the spectrum by overlaying the storage modulus with spectral values placed at half-decade increments in the frequency space, which was sufficient to accurately overlay the data. For a truly linear viscoelastic material, the spectral values as determined from the storage modulus should also reproduce the loss modulus through the second of eqns. (3), and for the low strain data, this is the case as observed below.

For dissipative materials like rubber compounds, the macro-mechanical properties – storage and loss moduli, including the effect of strain – are usually measured dynamically with a DMA (Dynamic Mechanical Analyzer). The dynamic moduli are usually determined by testing in a one-dimensional, strain-controlled mode, like uniaxial tension, pure bending or pure shear. Typical tests are performed in a cyclic mode, whether in tension, bending or shear, and consisted of sweeps through fixed frequencies of about $0.1\ \text{rad/s}$ to about $100\ \text{rad/s}$ and at fixed temperatures ranging from about $-70\ ^{o}C$ to about $+70\ ^{o}C$. The pure shear mode is accomplished by twisting a relatively thin and narrow specimen and is generally preferable to tensile or bending modes so as to minimize inertial effects at higher frequencies. This twisting mode of testing also has the advantage of producing a homogeneous strain field within the specimen, which is very important for testing at higher strain levels where all elements of the test cycle through the same strain levels. This would not be the case for bending modes, where the strain level in a bending cycle would vary with distance from the neutral axis.

For this study, a specimen (approximately 40x12.5x7 mm) of the rubber material taken from the backing of a typical belt material was sent to and tested by an independent laboratory [9] using a Rheometrics Scientific RDSII instrument to determine the low strain and higher strain viscoelastic properties in a twisting, pure shear mode. At low strain levels, testing was performed over a temperature range of $-70\ ^{o}C$ to $+75\ ^{o}C$ in about 10 degree increments and over frequencies of 0.0178 to $100\ \text{rad/s}$. Tests were also performed at various strain levels up to 6% at five temperatures and at a fixed frequency of $10\ \text{rad/s}$ to characterize the non-linear, strain dependent behavior.

Figure 1 shows the data of the test for the low strain moduli as taken through harmonic frequency sweeps at fixed temperatures and at strains well below the non-linear threshold (approximately 0.02%). Tests were also made at higher strains (up to 6%) to determine the effective moduli at these higher strain levels as will be discussed in Sec. 3.

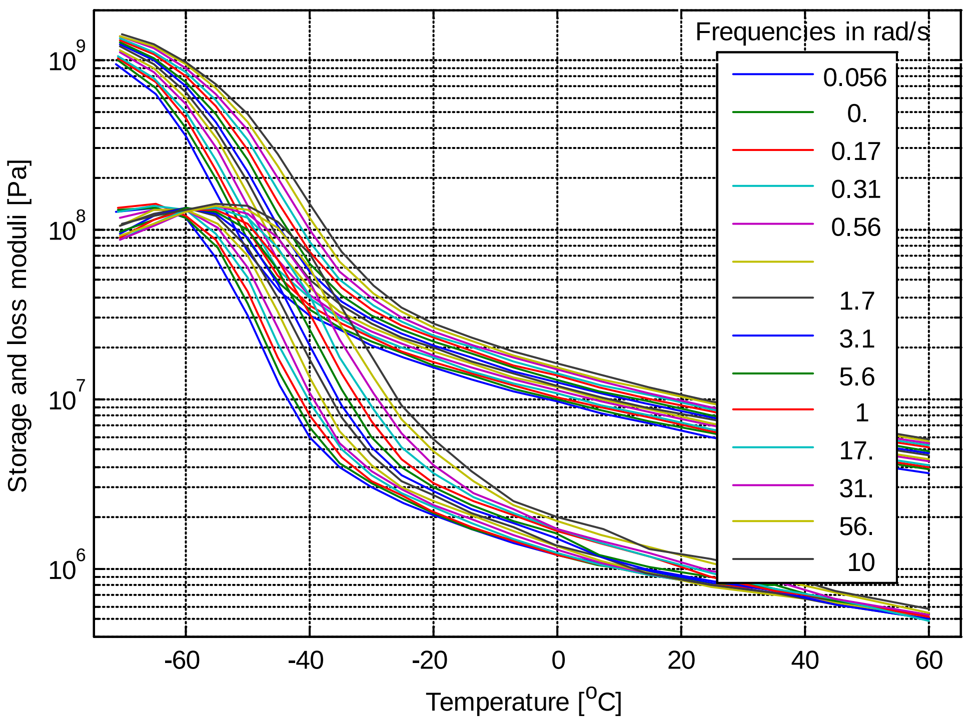

Fig. 1: Storage and loss moduli data for temperatures and frequencies tested.

Figures 2 and 3 show the master curves and shift parameter a_T for the rubber tested and developed at a reference temperature of $0^o C$, which is where a_T equals 1 of Figure 3. Shown also in the table of Figure 3 are the values of the WLF constants of eqn. (2) for this particular rubber compound.

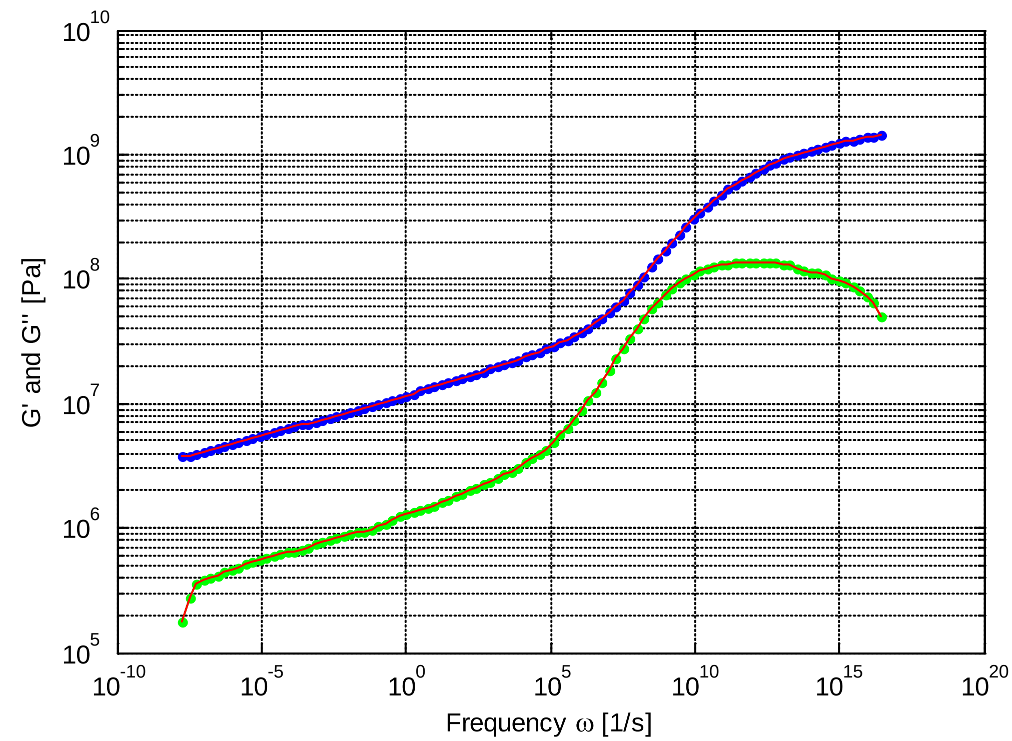

Fig. 2: Master curve of the shifted data and Prony series (spectral) fit to the shifted data.

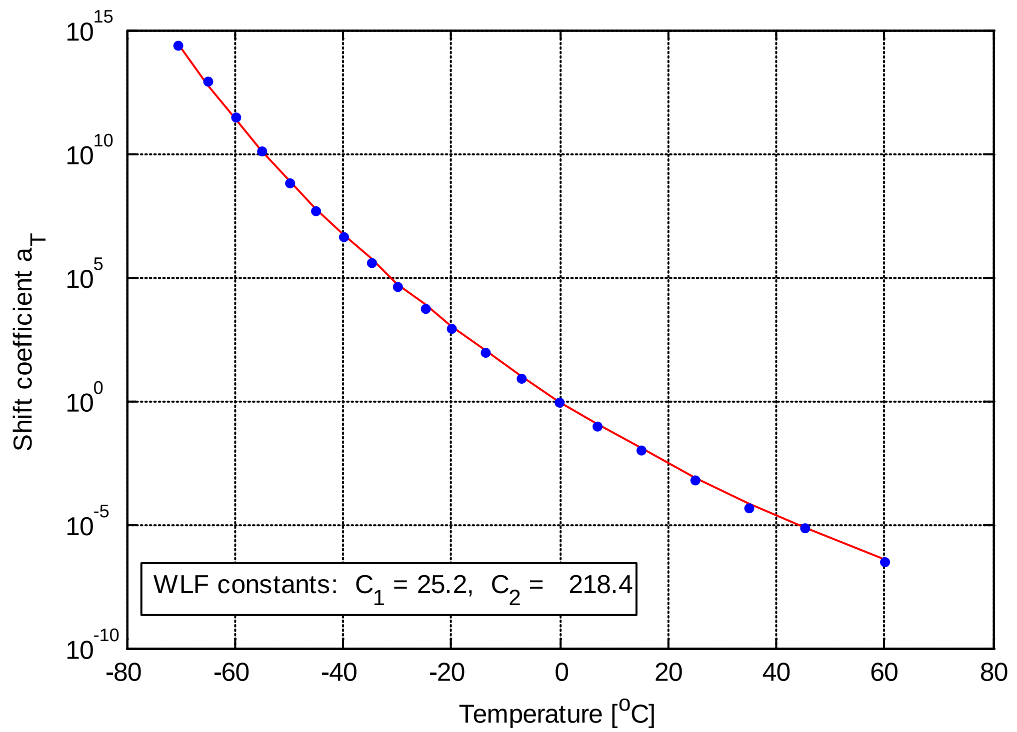

Fig. 3: Frequency-temperature shift coefficient a_T and WLF curve fit.

In addition to the moduli and the shift factor associated with the master curves, an important property directly related to the energy lost in a deformation cycle is the loss tangent $\tan(\delta) = E''/E' = G''/G'$, which for this material, is readily determined from Figure 2 and is shown in Figure 4 as a function of frequency. We observe that for this material there is a lower plateau of $\tan(\delta) \approx 0.1$ at the lower frequency range (higher temperatures) and peaks at nearly $\tan(\delta) \approx 0.5$ for a frequency of about 10^8 rad/s. As the rolling resistance of the belt backing over an idler is directly related to the energy absorbed by the backing material in the indentation deformation process, the loss tangent relates directly to the rolling resistance. Thus depending on the rate (frequency) and temperature, there could be as much as a fivefold variation in the indentation resistance for this material.

The above ideas pertain to a material characterized as a linear viscoelastic one, but can also form the basis for a nonlinear material by determination of approximately equivalent moduli and loss tangent, as discussed in the following section.

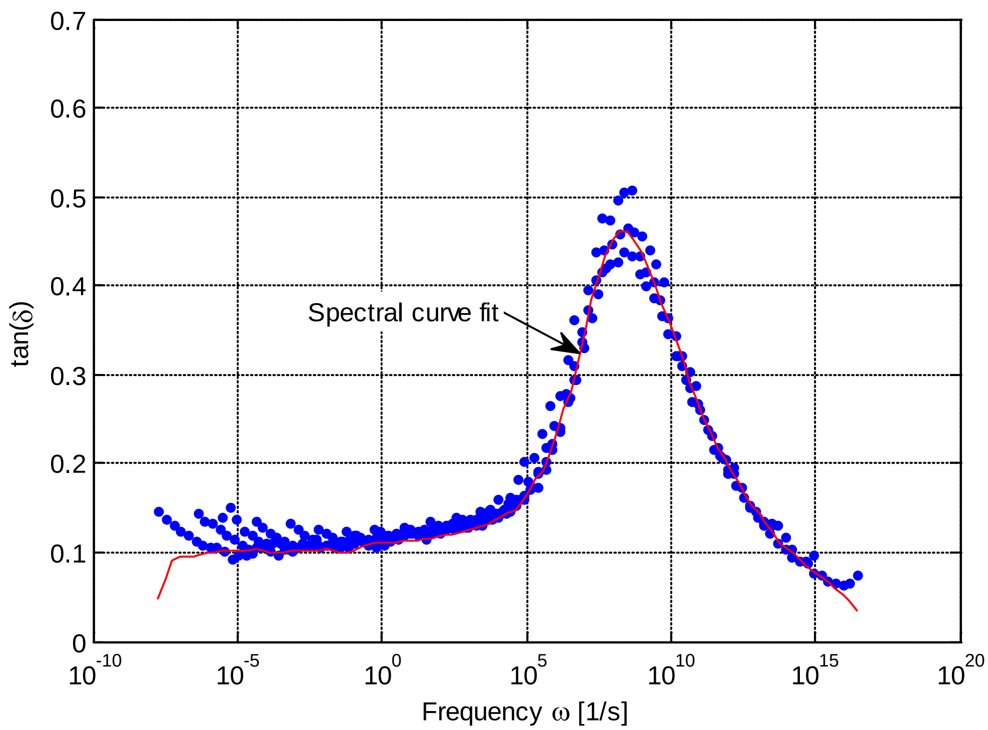

Fig. 4: Loss tangent and spectral representation of the data.

3. Strain Dependence of the Viscoelastic Moduli

Estimated compression strain levels into the backing material due to indentation can exceed 5 to 10% (Rudolphi [13]) and, at these strain levels, carbon black-filled rubber compounds are known to exhibit the nonlinear characteristic called the Payne effect, where the storage modulus decreases significantly with strain amplitude in cyclic, strain applied tests. As such, the linear relationship between stress and strain that defines the moduli in eqns. (1) and (3) is no longer strictly applicable. Also, in rubber compounds, the loss modulus generally increases with strain amplitude to a maximum at some strain level, and then decreases back to near the low strain plateau as observed by Vieweg, et. al. [6,7] and Wang [6]. To accurately predict the indentation resistance, we must then incorporate the nonlinear effects of strain amplitude into the stress response into the model of rolling resistance.

Kraus' [14] model of the Payne effect is a nonlinear constitutive equation that has been successfully applied to carbon-filled rubbers, and we follow that approach here since it is easily incorporated into simple models of the indentation and deformation of rolling contact. Kraus' model is phenomenologically based, but the macro-mechanical behavior is determined by a small number of parameters that can be determined by one-dimensional, cyclic strain tests. A one-dimensional test requires then a testing mode that is homogeneous, i.e., all elements of the specimen under test must go through the same stress/strain cycle.

Then, in addition to the frequency/temperature sweeps to determine the low strain moduli of previous section, additional tests were performed on the same material at five temperatures well above the glass transition temperature and at strains ranging from about 0.04% to 6% and all at a constant frequency - 10 rad/s. The data (moduli) from those tests are shown in Figure 5 as a function of strain at the five temperatures. As observed the storage modulus drops by nearly a factor of three from the low strain modulus to that at 6%, while the loss modulus increases slightly within that same strain range. There is also the expected rise in the loss modulus at strains in the range of about 1 to 6%. These test results are similar to those of Vieweg, et.al.[6,7], Wang [8] or Ulmer [4] and for which Kraus's model can be readily applied.

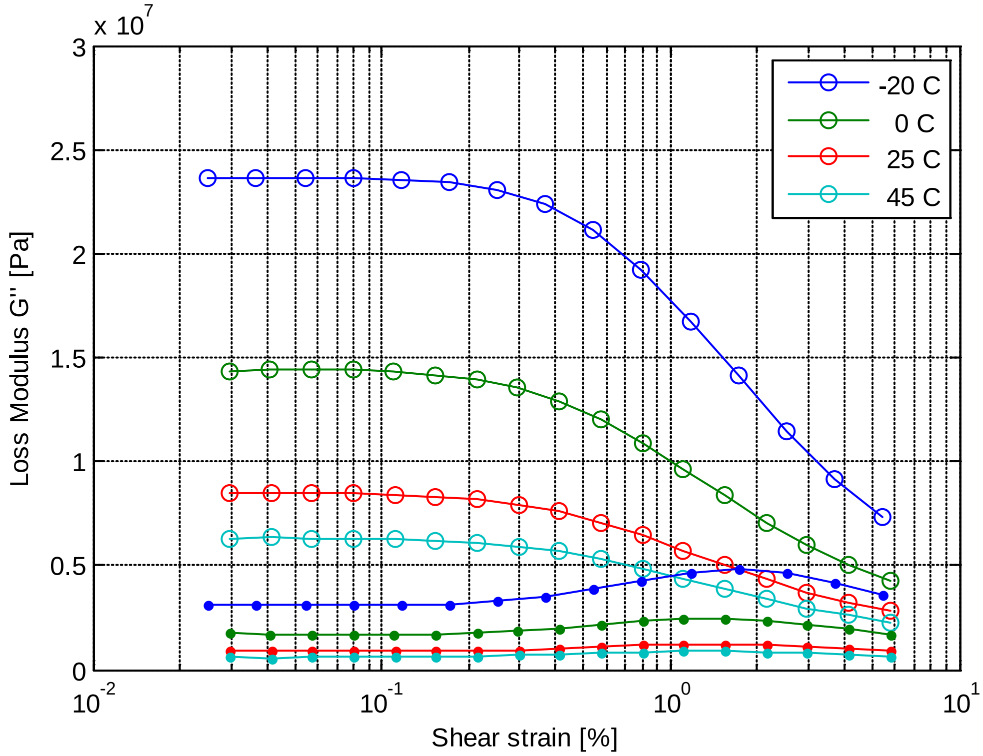

Fig. 5: Storage and loss moduli data at various temperatures.

A normalization of the data of Figure 5 is first determined by selection of a threshold or reference strain value for which the moduli plateau, which by inspection would be about $\gamma_0 = 0.1\%$. Division of the moduli of each temperature curve by the modulus at this strain, thus defining G_0' and G_0'' for each, produces Figure 6.

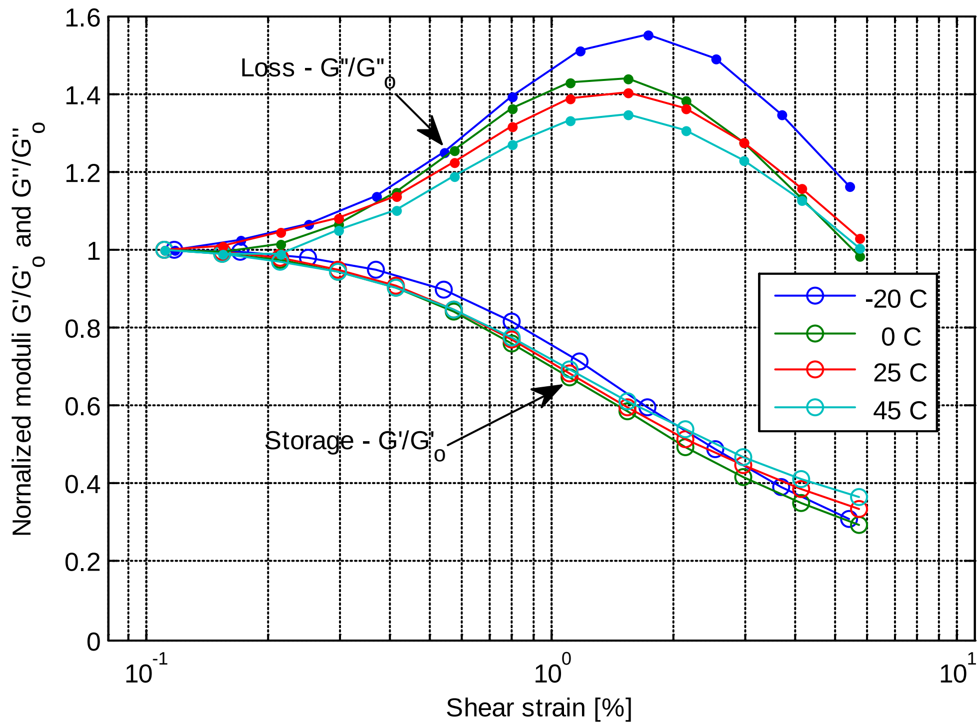

Fig. 6: Storage and loss moduli data normalized to the small strain values.

On Figure 6 we observe the fairly close overlay of the temperature curves, especially for the storage modulus. As such, it would not be inappropriate to presume that, for such materials, the moduli are nearly temperature, and therefore frequency, independent. This assumption is consistent with Vieweg, et.al. [6] for carbon filled rubbers, and we follow it here.

To fit the analytical model of Kraus to the data of Figure 6 we follow the notation and approach of Vieweg, et. al. [7] and Lion and Kardelky [15], where the nonlinear strain dependent moduli, deriving from Kraus' model, are written in the form,

$$G'(\gamma) = G'_\infty + \frac{(G'_0 - G'_\infty)}{1+(\gamma/\gamma_0)^{2m}}, \qquad G''(\gamma) = G''_\infty + \frac{(G''_0 - G''_\infty)(\gamma/\gamma_0)^m}{1+(\gamma/\gamma_0)^{2m}} \tag{4}$$

where G'_0 and G'_∞ are the storage moduli at small strains and large strain, respectively, and m is an adjustable parameter. By these equations, the Kraus model is a four-parameter model for each modulus, with the common parameter m. But Drozdov and Dorfmann [5] observe, amongst other things regarding the parameters, that while G'_0, G'_∞, etc., are fairly well predicted from experimental data in dynamics tests with fixed frequency, but that fitting data for the storage and loss moduli separately from eqns. (4) results in different values of the exponent m. They also observe that in fitting carbon-filled rubber data to the storage modulus results in a value of the exponent $m \approx 0.6$.

Thus in the fitting of these equations to the data of Figure 6 we allow a different m for each modulus.

Also, to fit the normalized data of Figure 6, we divide eqns. (4) by the low strain values G_0' and G_0'' and fit the data to equations of the form,

$$\overline{G'}(\gamma) = A + \frac{B}{1+(\gamma/\gamma_0)^{2m}}\,, \qquad \overline{G''}(\gamma) = A + \frac{B(\gamma/\gamma_0)^{m}}{1+(\gamma/\gamma_0)^{2m}} \tag{5}$$

where the constants A, B and m are determined independently for each modulus, as well as m. The low, threshold strain $\gamma_0 = 0.1\%$ is taken to be apply to both equations, so there are only three parameters for each modulus - A, B and m.

The data of Figure 6 were fit by a least squares process to eqns. (5) and Table 1 shows the resulting parameters. We observe that m is approximately 0.6 as noted previously by Drozdov and Dorfmann [5].

	A	B	m
Storage G'	0.1261	0.8990	0.5963
Loss G''	0.9894	0.9117	1.3414

Table 1: Curve fit parameters of Kraus' model for normalized storage and loss moduli

Figure 7 shows the analytical curve fits (red solid curves) of eqns. (5) for each modulus, overlaid with the data of Figure 6.

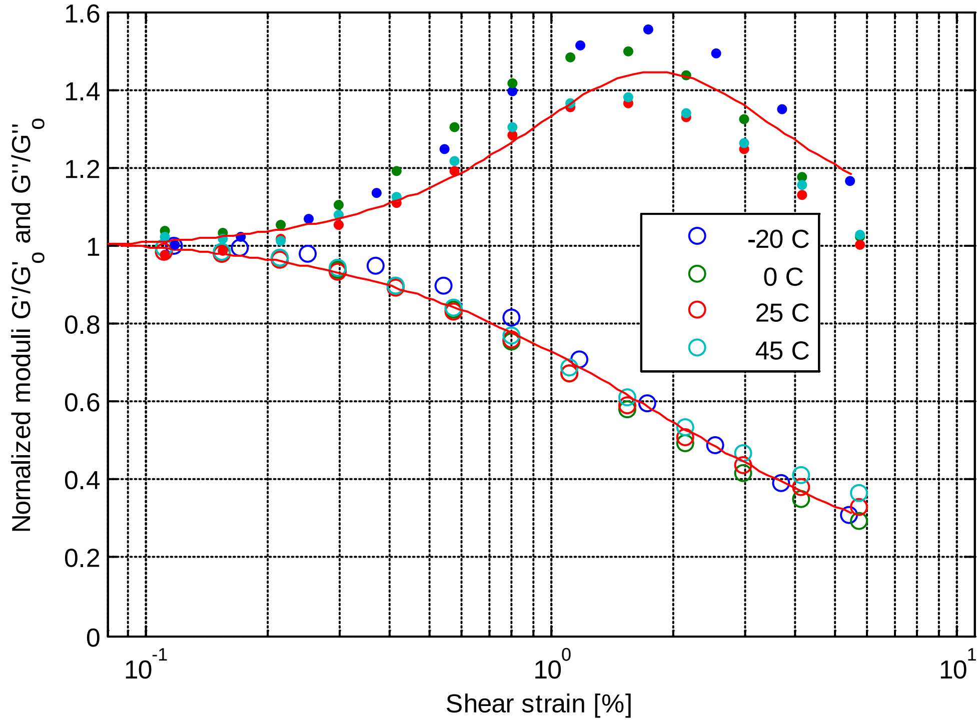

Fig. 7: Normalized storage and loss moduli with curve fits.

The parameter values of Table 1, together with eqns. (5) - (7) and (1) – (3) then constitute a complete one dimensional constitutive model for the material that includes temperature, rate (frequency) and strain level effects.

We also observe from Figure 7 that the storage modulus decreases to about one third of its low strain value at strains of about 6%. As will be seen in the following sections, this is a very important factor in indentation resistance.

Also, from a theoretical standpoint, at higher strain levels the assumption of linear viscoelasticity loses applicability and the characterization of material with parameters as above no longer applies. However, the above model for strain amplitude dependence does assume that, at higher strain levels, the material moduli are scaled up from the low strain level values according to Figure 7. In other words, at higher strain levels, the material is assumed to be a linear viscoelastic material with moduli determined from the values measured at low strain levels.

As a test of how well this scaling holds for higher strains, the same material was tested at a frequency of 10 rad/sec. and at various strain amplitudes, recording the stress and strain values at 100 points on each cycle of deformation. Figure 8 shows the results of this test at two strain amplitudes; a low and assumed to be linear strain of 0.1% (green loop) and a higher, and noticeably nonlinear strain of 3.2% (blue loop). At the lower strain level (green) the shape is elliptical while at the higher strain (blue), the Lissajous loop is asymmetric.

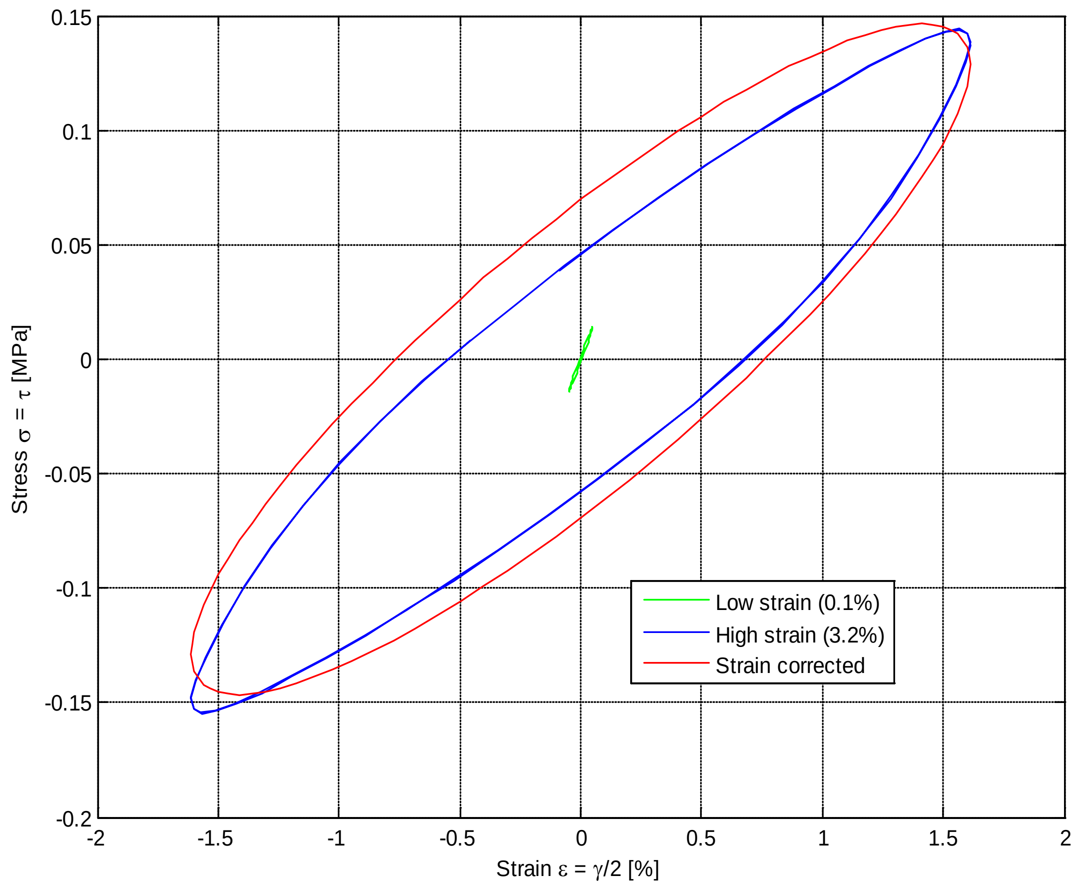

Fig. 8: Stress/strain cycles for low and high amplitude strains

Also shown in Figure 8 (red) is the strain corrected loop, as determined by the strain correction modification of the moduli according to Kraus' model. For this loop, the strain correction factors applied to the modulus G' is 0.427 and for G'' is 1.285, and which change the loss tangent $\tan(\delta) = G''/G'$ from 0.180 at low strain to 0.540 at high strains; a factor of 3.0. Thus by multiplication of the low strain moduli by these amounts, respectively, and assuming a strain amplitude of 3.2%, the green loop becomes the red loop. In principle, if the material were ideally linear viscoelastic, the red and blue loops would coincide and they would both be scaled up versions of the low stain (green) path with the same orientation of the axes of the ellipse.

Finally then, by Kraus' model, or the strain correction methodology used here, any strain corrected loop will remain elliptical, i.e., the material will be characterized as linear viscoelastic, but with alternate moduli. Since the energy dissipated per unit volume in a harmonic cycle of deformation of a viscoelastic material equals the area enclosed in the stress vs. strain graph, it is evident that this non-linear strain model is an approximation and may somewhat overestimate the energy dissipation.

4. Indentation deformation and resistance models

Analytical and computational models of rolling resistance due to indentation of the belt backing by idlers have been developed and, for design purposes, the influence of the main parameters such as idler diameter, carry weight, backing thickness and rubber properties of the backing have been well identified with simple analytical approaches. Simple models that treat the belt covering as a viscoelastic layer on a rigid base are that of Jonkers [16], Spaans [17] or Lodewijks [18]. A more rigorous approach, with the backing modeled as a full two-dimensional half-space, are those of May, et.al. [19], Hunter [20], Morland [21] or Goodier and Loutzenheiser [22]. All these models assume a linear viscoelastic material of the backing, and as shown by Lodewijks [18], the two-dimensional models provide slightly lower values of the indentation resistance for equal material properties and other system parameters.

One can also take a completely computational approach to rolling contact problems, as developed by Lynch [24] or Batra, et.al. [25], and as applied to belt cover indentation by Wheeler [26], where the cover deforms as a two-dimensional medium and is modeled by finite elements. The advantage of a computational approach is that a less restrictive deformation model is possible, such as modeling the entire belt carcass, as may be important for cable belts where the deformation between the steel cables also dissipate energy (cf. Wheeler [27]). Conversely, recourse to computational methods at the outset precludes analytical results that can make parameters explicit so that parameter studies important for design can be time consuming and expensive.

Experimental measurement of indentation resistance is possible, but known to be sensitive due to the low resistance force of efficient belt construction in laboratory setups or inaccessibility issues in field test.

The advantages and disadvantages of the various deformation models of the indentation process, or the preferred methodology of this calculation are not the focus here. We want to primarily show how the nonlinear constitutive equation affects the predicted indentation results. To do this it is only important to use a method where the strain dependent moduli can be incorporated without invalidating the assumption of the model itself. To this end we focus on the one-dimensional approaches where a simple one dimensional constitutive equation as derived from the test data can be merged directly into the indentation resistance model, as opposed to the two dimensional methods where the strain field produced by a rolling process would be inherently non-homogeneous and require unique strain compensated properties at each point. Instead we concentrate only on the models of Jonkers [16] and Rudolphi and Reicks [23], which is a direct extension of Lodewijks [18] for a multi-parameter material.

In the case of Jonkers, the equation for the indentation resistance factor f (ratio of belt resistance to vertical load, per unit belt width) is,

$$f = \left(\frac{Wh}{D^2}\right)^{1/3} \left(\frac{\pi}{2}\right) \frac{\tan(\delta)}{E'^{1/3}} \left[\frac{(\pi + 2\delta)\cos(\delta)}{4\sqrt{1+\sin(\delta)}}\right]^{4/3} \tag{8}$$

where W is the carry load per unit width, h is the backing thickness, E' is the storage modulus, D is the idler diameter and $\tan(\delta)$ is the loss tangent. The formula (8) is revealing in that, apart from showing the explicit dependence of f on the parameters W,

h and D, it shows that f is proportional to $\tan(\delta)/E'^{1/3} = E''/E'^{4/3}$. Thus if the storage modulus E' decreases and E'' increases, as for materials that exhibit the Payne effect, the rolling resistance will increase directly with changes in both moduli.

The formula of Jonkers in eqn. (8) results from the modeling assumption that the deformation process of indentation is harmonic, i.e., that the material of the backing continuously cycles through a compression phase of indentation, followed immediate by an equal tension phase. The indentation resistance f derives from the energy dissipation of this cyclic process and is thus half the area of the ellipse of Figure 7 with the strain amplitude determined from equilibrium of the carry load W with the backing stiffness, which requires a short iterative process to effectively determine the indentation depth of the idler, hence strain, for a given W. The assumed harmonic deformation process overestimates the actual case, however, since for steady belt speeds and uniform distance between idlers, the deformation is periodic, but not harmonic cycles as assumed by Jonkers.

The method of Lodewijks [18] circumvents the cyclic deformation process of Jonkers and directly determines the contact stress in the transient compression period of contact between the idler and backing. The rolling resistance then results from direct calculation of the moment of that stress about the idler center, hence equivalent force resisting the motion of the belt. Lodewijks developed this method for a simple three parameter material ($N = 1$ in eqn. (3)) and thus is not well suited for realistic rubber materials where $\tan(\delta)$ is spread over a frequency range as in Figure 4. Rudolphi and Reicks [23] generalized Lodewijks' approach by taking the material model to be an n-parameter Maxwell solid while retaining the same deformation assumption of the Winkler foundation. Their formula for the indentation resistance coefficient can be put into a form similar to the Jonkers' formula of eqn. (8), and is,

$$f = \left(\frac{Wh}{D^2}\right)^{1/3} \frac{M(E_i, \tau_i, \varsigma)}{F(E_i, \tau_i, \varsigma)^{4/3}} \tag{9}$$

where the functions $M(E_i, k_i, \varsigma)$ and $F(E_i, k_i, \varsigma)$ are defined by

$$M = \frac{E_0}{8}\left(1 - 2\varsigma^2 + \varsigma^4\right) + \sum_{i=1}^{N} E_i k_i \left[k_i^3 - \frac{k_i}{2}\left(1 + \varsigma^2\right) + \frac{1}{3}\left(1 + \varsigma^3\right) - k_i\left(1 + k_i\right)\left(k_i + \varsigma\right)e^{-(1+\varsigma)/k_i} \right]$$

and

$$F \equiv \frac{E_0}{6}\left(2 + 3\varsigma - \varsigma^3\right) + \sum_{i=1}^{N} E_i k_i \left\{ \frac{1-\varsigma^2}{2} - k_i\left[\left(1 + k_i\right)\left(1 - e^{-(1+\varsigma)/k_i}\right) - \left(1 + \varsigma\right)\right] \right\}.$$

In these definitions, E_i is the spectral stiffness of eqn. (3), $k_i = (v/a)\eta_i/E_i$ is the wave numbers associated with each spectral element, v is the belt speed, $a + b$ is the contact length and $\varsigma = b/a$, with a being the contact distance ahead of the idler center and b that behind. As with the Jonkers' formula (8), the contact length, hence strain, is determined by an iterative process of equilibrium, given the carry load W, with the stiffness of the backing.

Formulas (8) and (9) provide two different ways to determine the indentation resistance, based on the belt system parameters W, R, h, v and the material parameters E_i and τ_i. Both can be readily used in conjunction with eqns. (4) and (5) of Sec. 3 to allow for nonlinear effect of strain amplitude on the stress response of the backing material. They provide somewhat different results due to inherent assumptions of each, but both are used in the following section as a representative way to show the effects of strain amplitude on the indentation resistance calculation.

5. Indentation Resistance as Influenced by Strain Level

To show the effect of strain levels on the indentation resistance, the material properties of the tested material and material properties of Sec. 3 are combined with the two methods of Sec. 4. In the following calculation, the various belt parameters were taken as: $R = 0.075\,\text{m}$, $h = 0.00635\,\text{m}$, $v = 5.0\,\text{m/s}$, $T = 25\,^o\text{C}$ and we let W range from 50 to 4000 n/m.

Since strain in the indentation zone is largely determined by the carry load, we present results as W. Shown then in Figure 9 are the calculated values of f vs. W from both formulas (7) and (8), using both strain adjusted properties and properties based on small strains.

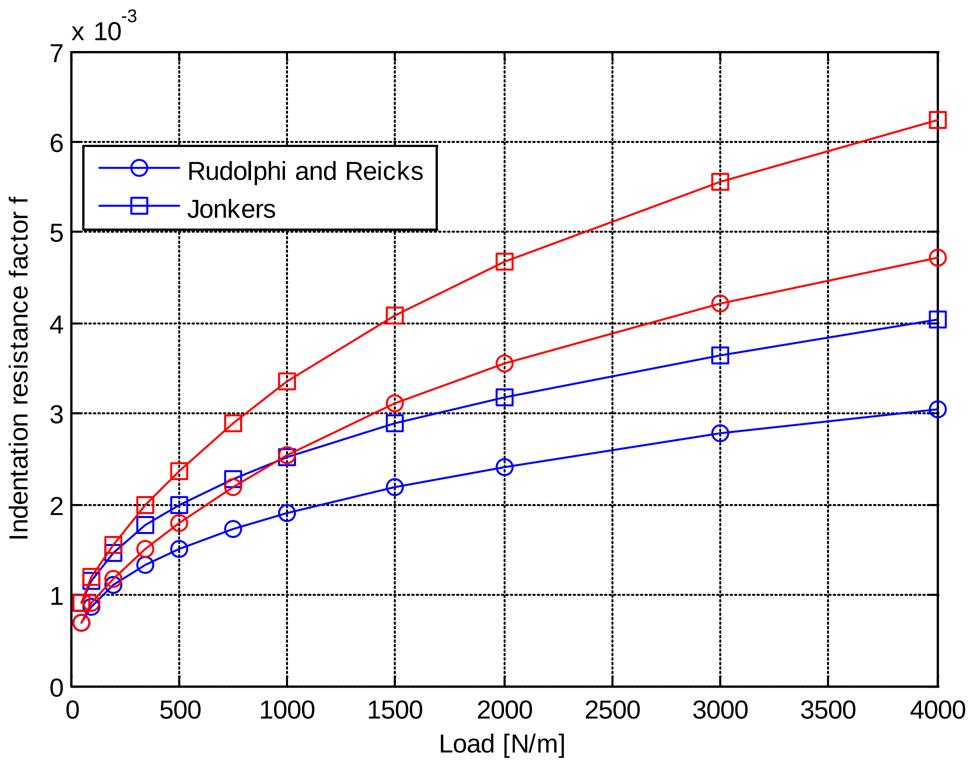

Fig. 9: Indentation resistance factors for small strain and strain-adjusted properties.

As may be seen, there is a difference in the indentation resistance based on the method of calculation, but both are magnified similarly with respect to the load W, or strain. The departure from the non-strain adjusted values increases with strain levels, or load, and converge at lower strains. For this particular material, there is nearly a factor of two at higher strain levels, i.e., the strain adjusted properties have a dramatic effect on the indentation resistance calculation, regardless of the methods used to make the calculation. Furthermore, this effect would be expected from Jonkers' method of eqn. (8) and the effect of strain level on the material moduli as seen in Figure 7.

6. Summary and Conclusions

As has been long known, the storage and loss moduli of particle filled elastomers, such as carbon black filled rubber as is commonly used for conveyor belt backing material, is dependent on strain levels experienced by the material. Large conveyor belt designs and applications, due to heavy carry loads, strain the backing well beyond the linear strain levels. By using Kraus' nonlinear constitutive model with measured data for a typical rubber and simple rolling resistance deformation models, we have shown that the predicted indentation resistance is highly sensitive to the material constitutive parameters, or strain level.

For the particular rubber properties used in this study, which is considered a typical belt backing material, these results would not be unexpected, considering the direct dependence of the loss tangent $\tan(\delta)$ on the moduli and the strong dependence of the indentation rolling resistance on the loss tangent. Hence realistic predictive methods for indentation prediction should include the nonlinear effects of strain, or values of indentation resistance, hence power required to drive the belt, could be seriously under-predicted.

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